King Fahd University of Petroleum & Minerals

Department of Information and Computer Science

Sample Solution

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
CLO	1	1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3	2	2	2	
Max	5	5	5	5	5	5	5	5	4	4	4	3	3	3	3	3	3	10	10	10	**100**
Earned																					

Question 1: [5 Points]

What is the contrapositive of the conditional statement:

"If it is hot, I will go swimming"?

If I will not go swimming, it is not hot.

Question 2: [5 Points]

Show that if $p \oplus p$ is a tautology, a contradiction, or a contingency.

p	p	$p \oplus p$
F	**F**	**F**
T	**T**	**F**

It is a contradiction

Question 3: [5 Points]

Are these system specifications consistent? Why? Why not?

a. If students can access the file system, then they can save new files.
b. Whenever the operating system is being upgraded, students cannot access the file system.
c. If students cannot save new files, then the operating system is not being upgraded.

Let

p = "students can access the file system"

q = "they can save new files"

r = "the operating system is being upgraded"

We have

a. $p \to q$ b. $r \to \neg p$ c. $\neg q \to \neg r$

These specifications are consistent if we find values for p, q, and r that makes the system true.

$(p \to q) \wedge (r \to \neg p) \wedge (\neg q \to \neg r)$

When p and r are false. In addition, when p is false, q is true and r true. Also when p and q are true and r is false.

p	q	r	$(p \to q) \wedge (r \to \neg p) \wedge (\neg q \to \neg r)$
F	F	F	T
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	T
T	T	T	F

Question 4: [5 Points]

Recall inhabitants of the island of knights and knaves. You encounter two people, A and B. A says "B is a knave" and B says "The two of us are both knights." Determine what A and B are.

A is a knight and B is a knave.

Question 5: [5 Points]

Show that if $(\neg s \land (r \rightarrow s)) \rightarrow \neg r$ *is* a tautology, a contradiction, or a contingency.

r	s	$(\neg s \land (r \rightarrow s)) \rightarrow \neg r$
F	F	T
F	T	T
T	F	T
T	T	T

It is a tautology

Question 6: [5 Points]

Determine whether the compound proposition $(r \rightarrow s) \land (r \rightarrow \neg s) \land (\neg r \rightarrow s) \land (\neg r \rightarrow \neg s)$ is satisfiable.

r	s	$(r \rightarrow s) \land (r \rightarrow \neg s) \land$ $(\neg r \rightarrow s) \land (\neg r \rightarrow \neg s)$
F	F	F
F	T	F
T	F	F
T	T	F

As there is no combinations of r and s that makes the proposition true, it is not satisfiable.

Question 7: [5 Points]

The domain of the propositional function $Q(x)$ consists of the integers 1, 2, 3, 4, and 5. Express the statement

$\forall x\, ((x \neq 3) \rightarrow Q(x)) \lor \exists x\, \neg Q(x)$

without using quantifiers, instead using only negations, disjunctions, and conjunctions.

$Q(1) \land Q(2) \land Q(4) \land Q(5) \lor \neg Q(1) \lor \neg Q(2) \lor \neg Q(3) \lor \neg Q(4) \lor \neg Q(5)$

Question 8: [5 Points]

Translate the statement:

"Nothing is in the correct place and is in excellent condition"

into a logical expression using predicates, quantifiers, and logical connectives.

Let $C(x)$ = "x is in the correct place" and $E(x)$ = "x is in excellent condition"

$\forall x\, \neg(C(x) \wedge E(x))$

Or

$\neg\exists x\, (C(x) \wedge E(x))$

Or equivalents.

Question 9: [4 Points]

Let $P(x)$ be "x is a baby" and $Q(x)$ be "x is logical". Suppose that the domain consists of all people. Express the statement "Babies are illogical" using quantifiers; logical connectives; and $P(x)$ and $Q(x)$.

$\forall x\, (P(x) \to \neg Q(x))$

Question 10: [4 Points]

Suppose the domain of $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Express the statement $\exists x \forall y\, P(x, y)$ using disjunctions and conjunctions.

$(P(1,1) \vee P(2,1) \vee P(3,1)) \wedge (P(1,2) \vee P(2,2) \vee P(3,2)) \wedge (P(1,3) \vee P(2,3) \vee P(3,3))$

Question 11: [4 Points]

Rewrite the following statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

$\neg\exists y(Q(y) \wedge \forall x \neg R(x, y))$.

$\forall y\, (\neg Q(y) \vee \exists x\, R(x, y))$.

Question 12: [3 Points]

What is the cardinality of $\{a, \{a\}, \{a, \{a\}\}\}$?

3

Question 13: [3 Points]

If A and B are sets then what is the value of $A \cap (B - A)$?

$\varnothing$

Question 14: [3 Points]

Is the following statement **TRUE** or **FALSE**?

$\{x\} \subseteq \{x\}$ and $\{x\} \in \{x\}$.

FALSE

Question 15: [3 Points]

Is the following statement **TRUE** or **FALSE**?

If $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$ then $C \times B \times A = C \times A \times B$.

FALSE

Question 16: [3 Points]

Is the following statement **TRUE** or **FALSE**?

If A and B are sets then $A - B \subseteq A$.

TRUE

Question 17: [3 Points]

Is the following statement **TRUE** or **FALSE**?

If A, B, and C are sets then $(A - B) - C \subseteq A - C$.

TRUE

Question 18: [10 Points]

Given the premises: $p \rightarrow (q \wedge r)$, $s \rightarrow r$, and $r \rightarrow p$.
show that $s \rightarrow q$.

[1] $p \rightarrow (q \wedge r)$ Premise
[2] $s \rightarrow r$ Premise
[3] $r \rightarrow p$ Premise
[4] $r \rightarrow (q \wedge r)$ Hypothetical syllogism of [3] & [1]
[5] $(r \rightarrow q) \wedge (r \rightarrow r)$ The equivalence of [4]
[6] $r \rightarrow q$ Simplification of [5]
[7] $s \rightarrow q$ Hypothetical syllogism of [2] & [6]

RULES OF INFERENCE	
p ------ $\therefore\ p \vee q$	*(Addition)*
$p \wedge q$ ------ $\therefore\ p$	*(Simplification)*
p q ------ $\therefore\ p \wedge q$	*(Conjunction)*
p $p \rightarrow q$ -------- $\therefore\ q$	*(Modus ponens)*
$\neg q$ $p \rightarrow q$ -------- $\therefore\ \neg p$	*(Modus tollens)*
$p \rightarrow q$ $q \rightarrow r$ -------- $\therefore\ p \rightarrow r$	*(Hypothetical syllogism)*
$p \vee q$ $\neg p$ ------ $\therefore\ q$	*(Disjunctive syllogism)*
$p \vee q$ $\neg p \vee r$ ------ $\therefore\ q \vee r$	*(Resolution)*

Question 19: [10 Points]

Prove that the difference of an even integer minus an odd integer is odd. Use direct proof.

Given $n = a - b$ where a is an even integer ($a = 2k$) and b is an odd integer ($b = 2m + 1$), we want to prove that n is odd.

$$
\begin{aligned}
n &= a - b, \text{ } a \text{ is even and } b \text{ is odd} \\
&= 2k - (2m + 1) \\
&= 2k - 2m - 1 \\
&= 2k - 2m - 1 + (1 - 1) \\
&= 2k - 2m - 2 + 1 \\
&= 2(k - m - 1) + 1 \\
&= 2j + 1 \qquad \text{where } j = k - m - 1
\end{aligned}
$$

So n is equal to $2j + 1$ where j is an integer number. This proves that n is odd.

Question 20: [10 Points]

Using membership table, Prove that $(A_1 \cap A_2) \cup B = (A_1 \cup B) \cap (A_2 \cup B)$, for every sets A_1, A_2, B.

A_1	A_2	B	$(A_1 \cap A_2)$	$(A_1 \cap A_2) \cup B$	$(A_1 \cup B)$	$(A_2 \cup B)$	$(A_1 \cup B) \cap (A_2 \cup B)$
0	0	0	0	0	0	0	0
0	0	1	0	1	1	1	1
0	1	0	0	0	0	1	0
0	1	1	0	1	1	1	1
1	0	0	0	0	1	0	0
1	0	1	0	1	1	1	1
1	1	0	1	1	1	1	1
1	1	1	1	1	1	1	1

The membership table for these combinations of sets is shown. This table has eight rows. Because the columns for $(A_1 \cap A_2) \cup B$ and $(A_1 \cup B) \cap (A_2 \cup B)$ are the same, the identity is valid.